HIGH-GAIN OBSERVER BASED STATE AND PARAMETER ESTIMATION IN NONLINEAR SYSTEMS

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Abstract: Motivated by constant parameter estimation in nonlinear models, as well as state observation in dynamical systems, an adaptive observer is proposed for a class of nonlinear systems linearly depending on unknown parameters. This observer is derived from the well-known high-gain nonlinear design on the one hand, and an adaptive linear one on the other hand, on the basis of previous results in that direction (Xu, 2002). Global exponential convergence is shown under appropriate excitation conditions, and the results are illustrated by simulations.

Keywords: Nonlinear systems, nonlinear input-output models, nonlinear observers, adaptive observers, high gain.

1. INTRODUCTION

The problem of parameter estimation in nonlinear systems has motivated a lot of work, for system identification or fault detection for instance.
In practise one generally has to additionally cope with the lack of measurements on internal dynamics, which amounts to a problem of state reconstruction. In many cases, high gain techniques proved to be very efficient for state estimation, leading to the now well-known *high gain observer* (Gauthier *et al.*, 1992).
When the system further depends on some unknown parameters, the observer design has to be modified so that both state variables and parameters can be estimated, leading to so-called *adaptive observers*. Various results in that respect can be found, going back to (Lüders and Narendra, 1973; Carroll and Lindorff, 1973; Kreisselmeier, 1977) for linear systems, or (Bastin and Gevers, 1988; Marino, 1990) for nonlinear ones, but with nonlinearities *only depending on the measured input/output*. Some extensions to more general cases have been proposed under some particular "passivity-like" condition (Cho and Rajamani, 1997; Besançon, 2000; Besançon and Zhang, 2002) or using a variable structure approach (Martinez and Poznyak, 2001).
The purpose of the present note is to show how high gain techniques can also be useful in the context of state and parameter estimation.
Recently indeed, on the basis of a new result on adaptive observation for linear time-varying systems (Zhang, 2002), an adaptive observer has been proposed for a class of nonlinear systems admitting some high gain observer, but further depending on unknown parameters (Zhang *et al.*, 2003). In that previous study, the unknown parameters enter the system through some known time-varying vector fields. One contribution of the present note is to extend this result to the case when those vector fields can depend on the whole state, namely *the coefficients of the parameters are not directly known*. The assumptions on the considered

class of systems are basically that if all the parameters were known, some high-gain observer could be designed in a classical way, and that the systems are 'sufficiently excited' in a sense which is close to usually required assumptions on adaptive systems [1] (signals must be rich enough so that the unknown parameters can indeed be identified). The result is then extended to a class of *multi-output* nonlinear systems, which in particular includes possible state-space representations of linearly parameterized input-output models. This makes it of particular interest for parameter identification in nonlinear systems.

The considered class of systems and its motivation are presented in section 2, and the corresponding proposed estimation scheme for both the state and the parameter vectors is given in section 3. Some examples are presented in section 4 as an illustration of the proposed results, and conclusions end the paper in section 5.

2. PROBLEM FORMULATION AND MOTIVATION

2.1 Background results on high-gain observers

High-gain observer design is related to systems of specific structure (Gauthier *et al.*, 1992), roughly corresponding to the property for the system to be observable for any input (Gauthier and Bornard, 1981).
In short, this structure is as follows:

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + \varphi(x(t), u(t)) \\ y(t) &= C_0 x(t)\end{aligned} \tag{1}$$

where $x \in I\!R^n$ denotes the state vector, $u \in I\!R^m$ is the input vector, and $y \in I\!R$ the measured output, while A_0, C_0 and φ satisfy:

$$\begin{aligned}A_0 &= \begin{pmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ & & & 1 \\ 0 & & & 0 \end{pmatrix}, \\ C_0 &= \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \quad \text{and}\end{aligned} \tag{2}$$

$$\varphi(x,u) = \begin{pmatrix} \varphi_1(x_1,u) & \varphi_2(x_1,x_2,u) & .. & \varphi_n(x,u) \end{pmatrix}^T \tag{3}$$

if $x = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \end{pmatrix}^T$.
Using some Lipschitz property of φ in x uniformly in u, an observer for (1) can be designed as follows (Gauthier *et al.*, 1992):

$$\dot{\hat{x}} = A_0\hat{x} + \varphi(\hat{x}, u) - \lambda\Lambda(\lambda)^{-1}K_0(C_0\hat{x} - y) \tag{4}$$

where K_0 is such that $A_0 - K_0C_0$ is stable, $\Lambda(\lambda)^{-1} = diag(1, \lambda, \lambda^2 ... \lambda^{n-1})$, and $\lambda > 0$ is chosen large enough.

Notice that such high-gain techniques also apply to multi-output systems (Bornard and Hammouri, 1991), whose characterization has been recently revisited (Bornard and Hammouri, 2002).
In the case of two outputs for instance, a simple structure for which observer (4) can easily be extended is as follows (Besançon and Hammouri, 1998):

$$\begin{aligned}\dot{x}(t) &= \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} x(t) + \begin{pmatrix} \varphi_1(x(t), u(t)) \\ \varphi_2(x(t), u(t)) \end{pmatrix} \\ y(t) &= \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} x(t)\end{aligned} \tag{5}$$

where A_1, A_2 (resp. C_1, C_2) are of the same form as A_0 (resp. C_0) in (2), and if $x = \begin{pmatrix} x[1] \\ x[2] \end{pmatrix}$ with $x[i]$ of same dimension as A_i, functions φ_1, φ_2 satisfy the same structure as φ in (3) w.r.t. $x[i]$ respectively, but with their respective last components which can additionally depend on $x[j], j \neq i$.
In that case indeed, one can design an observer for (5) by combining observers of the form (4) respectively designed for $x[1]$ and $x[2]$ (Besançon and Hammouri, 1998).

2.2 A class of parameter-affine systems

The problem we consider here is that of state *and parameter* estimation for multi-output systems of a structure like (5), which can be described as follows:

$$\begin{aligned}\dot{x}(t) &= \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} x(t) + \begin{pmatrix} \varphi_1(x(t), u(t)) \\ \varphi_2(x(t), u(t)) \end{pmatrix} \\ &\quad + \begin{pmatrix} \psi_1(x,u) \\ \psi_2(x,u) \end{pmatrix} \theta \\ y(t) &= \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} x(t)\end{aligned} \tag{6}$$

where $\theta \in I\!R^q$ is a vector of unknown parameters, and ψ_1, ψ_2 are $n_i \times q$ matrices (for some n_1, n_2) satisfying:

$$\psi_i(x,u) = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \\ \psi_{in_i,1}(x,u) & \cdots & \psi_{in_i,q}(x,u) \end{pmatrix} \tag{7}$$

Considering such a class of state-space representation subject to unknown parameters first extends previous results on adaptive high gain observers (Zhang *et al.*, 2003), obtained for single output systems and when $\psi(x,u) = \psi(t)$ is known.
It is further motivated by the fact that a general nonlinear input-output model of a (siso) system linear in unknown parameters of the following form [2] :

$$\begin{aligned}y^{(n)} &= \Phi_0(y^{(n-1)}, \ldots, \dot{y}, y, u^{(m)}, \ldots, \dot{u}, u) \\ &\quad + \Phi_1(y^{(n-1)}, \ldots, \dot{y}, y, u^{(m)}, \ldots, \dot{u}, u)^T\theta\end{aligned} \tag{8}$$

[1] see e.g. (Sastry and Bodson, 1989)

[2] Here $v^{(k)} = \frac{d^k}{dt^k}v$ for any integer k and any function of time $v(t)$

can be re-written as (6) for instance whenever the input function u satisfies (as in (Ciccarella *et al.*, 1993)):

$u^{(r+1)}(t) = 0$ almost everywhere for $t \geq 0$
and some $r \in I\!N$,

which happens e.g. for piecewise polynomial input functions.
Just consider indeed the time derivatives of y as the state variables corresponding to $x[1]$, and the time derivatives of u as those corresponding to $x[2]$, with y and u as the output variables: then we clearly end up with a state-space representation of the form (6).
Notice also that input-output regressions which are linear in the parameters - as in (8) - have been shown to roughly characterize *globally identifiable* models (Ljung and Glad, 1994).

3. ADAPTIVE OBSERVER DESIGN

3.1 Single output case

Let us first consider a system of the following form:

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + \varphi(x(t), u(t)) + \psi(x(t), u(t))\theta \\ y(t) &= C_0x(t) \end{aligned} \quad (9)$$

where $x \in I\!R^n$ denotes the state vector, assumed to start in $\chi_0 \subset I\!R^n$, $u \in I\!R^m$ is the input vector, $y \in I\!R$ the measured output, and $\theta \in I\!R^q$ a vector of unknown parameters, i.e. $\psi \in I\!R^{n\times q}$.
Let us assume that a high gain observer can be designed as in (4) whenever θ is known, by considering that the following conditions hold:
$[A1]$ A_0, C_0, φ are as in (2)-(3), and $\psi(x, u)$ is an $n\times q$ matrix of the same form as ψ_i in (7).
$[A2]$ φ, ψ are smooth functions w.r.t. their arguments, and u is bounded generating bounded states $\|x(t)\| \leq X$ for any $x(0) \in \chi_0$, while $\|\theta\| \leq \Theta$.

The main result of this section is that under some additional condition of *persistent excitation* - as usually required in adaptive systems - an adaptive version of the available high gain observer can be designed:

$[A3]$ Given some K_0 making $A_0-K_0C_0$ to be a stable matrix, inputs u are such that the state vector satisfies the following property:
for any $x(0) \in \chi_0$, and any $\Gamma(0) \in I\!R^{n\times q}$, the solution $\Gamma(t)$ of:

$$\dot{\Gamma}(t) = \lambda(A_0 - K_0C_0)\Gamma(t) + \lambda\psi(x(t), u(t)) \quad (10)$$

is such that for some $t_0 \geq 0$:

$$\begin{aligned} &\exists\alpha, T \text{ independent of } \lambda \ : \ \forall t \geq t_0, \\ &\text{and for } \lambda \text{ large enough,} \\ &\int_t^{t+T} \Gamma(\tau)^T C_0^T C_0\Gamma(\tau)d\tau \geq \alpha I \end{aligned} \quad (11)$$

where I stands for the identity.

Remark *3.1.* Assumption [A3] to some extent corresponds to $\psi(x(t), u(t))$ being persistently exciting, since $C_0\Gamma$ is obtained by filtering ψ through a linear stable minimum phase transfer.
The fact that α, T must be independent of λ can for instance hold whenever the limiting Γ when $\lambda \to \infty$ (namely $-[A_0 - K_0C_0]^{-1}\psi$) satisfies the inequality of (11).

The result is as follows:

Theorem *3.1.* Given a system (9) satisfying assumptions $[A1], [A2], [A3]$, for λ large enough, the system below is an asymptotic observer for (9), in the sense that for any initial condition $x(0) \in \chi_0$ and any $\hat{\theta}(0)$, $\hat{x}(0)$ respectively bounded by Θ and X, $\|\hat{x}(t)-x(t)\|$ and $\|\hat{\theta}(t) - \theta\|$ exponentially go to zero:

$$\begin{aligned} &\dot{\hat{\Gamma}}(t) = \lambda(A_0 - K_0C_0)\hat{\Gamma}(t) + \lambda\psi(\tilde{x}(t), u(t)) \\ &\dot{\hat{x}}(t) = A_0\hat{x}(t) + \varphi(\tilde{x}(t), u(t)) + \psi(\tilde{x}(t), u(t))\tilde{\theta}(t) \\ &\qquad + \Lambda(\lambda)^{-1}[\lambda K_0 + \hat{\Gamma}(t)\hat{\Gamma}^T(t)C_0^T][y(t) - C_0\hat{x}(t)] \\ &\dot{\hat{\theta}}(t) = \lambda^n\hat{\Gamma}(t)^T C_0^T [y(t) - C_0\hat{x}(t)] \\ &\quad \tilde{x} = \hat{x} \ if \ \|\hat{x}\| \leq X, \ \frac{\hat{x}}{\|\hat{x}\|}X \ otherwise, \\ &\quad \tilde{\theta} = \hat{\theta} \ if \ \|\hat{\theta}\| \leq \Theta, \ \frac{\hat{\theta}}{\|\hat{\theta}\|}\Theta \ otherwise. \end{aligned} \quad (12)$$

where K_0 and $\Lambda(\lambda)$ are as in (4).

The proof is based on the following lemmas:

Lemma *3.1.* $\forall\mu > 0$, $\forall t_1 \geq 0$ $\exists\lambda_1 > 0$ such that $\forall\lambda \geq \lambda_1$, $\hat{x}$ given by (12) satisfies:

$$\|\hat{x}(t) - x(t)\| \leq \mu \ \forall t \geq t_1.$$

In other words, whatever θ is (satisfying $[A2]$), observer (12) can provide arbitrarily accurate estimation of x by appropriately tuning θ (see e.g. (Besançon, 2003)).

Lemma *3.2.* If $[A3]$ holds and $\hat{\Gamma}$ is given by (12), then $\forall t_2 \geq 0$, $\exists\mu > 0$, $\exists\lambda_2 > 0$ such that $\forall\lambda \geq \lambda_2$:

$$\begin{aligned} &\|\hat{x}(t) - x(t)\| \leq \mu, \text{ for } t \geq t_2 \\ &\quad \Rightarrow \ \hat{\Gamma} \text{ satisfies (11) for } t \geq t_2. \end{aligned}$$

This means that for some estimation $\hat{x}$ of x accurate enough (which can always be obtained in view of lemma 3.1), $\hat{\Gamma}$ satisfies the same excitation property as Γ.

Lemma *3.3.* If $\hat{\Gamma}$ given by (12) satisfies (11), then there exists $\lambda_3 > 0$ such that for all $\lambda \geq \lambda_3$ in (12), $\varepsilon_\theta := \frac{1}{\lambda^n}(\hat{\theta} - \theta)$ and $\bar{\varepsilon} := \Lambda(\lambda)[\hat{x} - x] - \hat{\Gamma}\varepsilon_\theta$ exponentially go to zero.

Lemma 3.3 clearly states that if $\hat{\Gamma}$ is sufficiently exciting (which can be made to be true by choosing λ large enough in view of previous lemmas), then the result of theorem 3.1 is indeed achieved.

The proofs of lemmas 3.1, 3.2, 3.3 are as follows:

• *Proof of lemma 3.1:* Set $\varepsilon_0 := \Lambda(\lambda)[\hat{x} - x]$, then one can check that:

$$\dot{\varepsilon}_0 = \lambda(A_0 - K_0C_0)\varepsilon_0 + \Lambda(\lambda)\Delta(\varphi) + \Lambda(\lambda)\Delta(\psi)\theta + \hat{\Gamma}\hat{\Gamma}^T C_0^T C_0 \varepsilon_0 + \Lambda(\lambda)\psi(\tilde{x}, u)(\tilde{\theta} - \theta)$$

where

$$\begin{array}{l}\Delta(\varphi) = \varphi(\tilde{x}, u) - \varphi(x, u)\\ \Delta(\psi) = \psi(\tilde{x}, u) - \psi(x, u).\end{array} \qquad (13)$$

Notice that from $[A2]$ and the definition of $\tilde{x}$, $\Delta(\varphi)$ and $\Delta(\psi)$ are bounded, and from $[A1]$:

$$\begin{array}{l}\|\Lambda(\lambda)\Delta(\varphi)\| \leq \gamma_\varphi \|\varepsilon_0\|,\\ \|\Lambda(\lambda)\Delta(\psi)\| \leq \gamma_\psi \|\varepsilon_0\|\end{array} \qquad (14)$$

for Lipschitz constants $\gamma_\varphi, \gamma_\psi$.
Notice also that from its definition, $\hat{\Gamma}$ is upper bounded independently of λ, and that from $[A2]$ and the definition of $\Lambda, \tilde{\theta}, \tilde{x}$, $\|\Lambda(\lambda)\psi(\tilde{x}, u)(\tilde{\theta} - \theta)\| \leq \frac{\delta}{\lambda^{n-1}}$ for some δ independent of λ.
Now by choosing $V_0 := \varepsilon_0^T P_0 \varepsilon_0$ with P_0 such that:

$$P_0(A_0 - K_0C_0) + (A_0 - K_0C_0)^T P_0 = -I, \quad (15)$$

one can check that:

$$\dot{V}_0 \leq -(\lambda - a)\|\varepsilon_0\|^2 + \frac{b}{\lambda^{n-1}}\|\varepsilon_0\|$$

where a, b are constants independent of λ, and from this $\|\varepsilon_0\|$ is made smaller than $\frac{b}{\lambda^{n-1}(\lambda - a)}$, which gives the result for $\|\hat{x} - x\|$.

• *Proof of lemma 3.2:* Set $E := \hat{\Gamma} - \Gamma$. Then:

$$\dot{E} = \lambda(A_0 - K_0C_0)E + \lambda\Delta(\psi)$$

Considering here, for each column E_i of E, a function $V_i := E_i^T P_0 E_i$ with P_0 as in (15), one can check that for $\|\hat{x} - x\| \leq \mu$, $\dot{V}_i \leq -\lambda\|E_i\|^2 + \lambda\delta\mu\|E_i\|$ for some δ independent of μ. Hence $\|E\|$ can become arbitrarily small according to μ. Moreover, notice that by definition Γ is upper bounded independently of λ, say by π.
Now considering the problem of $\hat{\Gamma} = \Gamma + E$ satisfying (11), we have:

$$\int_t^{t+T} \hat{\Gamma}(\tau)^T C_0^T C_0 \hat{\Gamma}(\tau) d\tau \qquad (16)$$

$$= \int_t^{t+T} \Gamma(\tau)^T C_0^T C_0 \Gamma(\tau) d\tau \qquad (17)$$

$$+ \int_t^{t+T} [E(\tau)^T C_0^T C_0 \Gamma(\tau) + \Gamma(\tau)^T C_0^T C_0 E(\tau)] d\tau \qquad (18)$$

$$+ \int_t^{t+T} E(\tau)^T C_0^T C_0 E(\tau) d\tau \qquad (19)$$

First, by $[A3]$, $(17) \geq \alpha I$, and clearly $(19) \geq 0$.
Then, if $\|E\| \leq \mu_E$ and $\|\Gamma\| \leq \pi$, we have:

$$\|(18)\| \leq \int_t^{t+T} \|E^T C_0^T C_0 \Gamma\| d\tau \leq \mu_E T \pi$$

and thus $(18) \geq -\mu_E T \pi I$.
Finally, $(16) \geq (\alpha - \mu_E T\pi)I$, and thus, for μ_E small enough, namely $\|\hat{x} - x\|$ small enough, (11) holds for $\hat{\Gamma}$.

• *Proof of lemma 3.3*: Set $\varepsilon_\theta := \frac{1}{\lambda^n}(\hat{\theta} - \theta)$ and $\varepsilon := \Lambda(\lambda)[\hat{x} - x] - \hat{\Gamma}\varepsilon_\theta$.
Then one can check that:

$$\begin{array}{rcl}\dot{\varepsilon} &=& \lambda(A_0 - K_0C_0)\varepsilon + \Lambda(\lambda)\Delta(\varphi) + \Lambda(\lambda)\Delta(\psi)\theta\\ \dot{\varepsilon}_\theta &=& -\hat{\Gamma}^T C_0^T C_0 \hat{\Gamma}\varepsilon_\theta - \hat{\Gamma}^T C_0^T C_0 \varepsilon\end{array} \qquad (20)$$

Now one has simply to choose some appropriate Lyapunov function for (20):
On the one hand, one can take again P_0 as in (15).
On the other hand, noting that if $\hat{\Gamma}$ satisfies (11), then $\dot{\varepsilon}_\theta = -\hat{\Gamma}^T C_0^T C_0 \hat{\Gamma}\varepsilon_\theta$ is classically exponentially stable (Narendra and Annaswamy, 1989), one can consider some positive definite matrix P satisfying:

$$\dot{P} = [\hat{\Gamma}^T C_0^T C_0 \hat{\Gamma}]^T P + P[\hat{\Gamma}^T C_0^T C_0 \hat{\Gamma}] - I.$$

One can check that $\hat{\Gamma}$ being upper bounded independently of λ, P classically satisfies $p_1 I \leq P(t) \leq p_2 I$ for any t and some p_1, p_2 independent of λ.
Finally one can choose $V(\varepsilon, \varepsilon_\theta) := \varepsilon^T P_0 \varepsilon + \varepsilon_\theta^T P \varepsilon_\theta$.
We indeed get:

$$\begin{array}{rcl}\dot{V} &=& -\lambda\varepsilon^T\varepsilon + 2\varepsilon^T P_0 \Lambda(\lambda)[\Delta(\varphi) + \Delta(\psi)\theta]\\ && -\varepsilon_\theta^T\varepsilon_\theta + 2\varepsilon_\theta^T P \hat{\Gamma}^T C_0^T C_0 \varepsilon\end{array}$$

and by using (14) and the fact that $\hat{\Gamma}$ and P are upper bounded independently of λ, for λ large enough we can easily obtain:

$$\dot{V} \leq -c\|\varepsilon\|^2 - d\|\varepsilon_\theta\|$$

for some $c, d > 0$, and thus, $\dot{V} \leq -\kappa V$ which gives the conclusion.

3.2 The two-output case

Now let us come to the case of a system (6) written in compact form as:

$$\begin{array}{rcl}\dot{x} &=& Ax + \varphi(x, u) + \psi(x, u)\theta\\ y &=& Cx\end{array} \qquad (21)$$

with $x = \begin{pmatrix} x[1] \\ x[2] \end{pmatrix}$, $x[i] = (\, x_1[i] \quad \ldots \quad x_{n_i}[i]\,)^T \in I\!R^{n_i}$, $y \in I\!R^2$, and A, C, φ, ψ given by (6).

Assumptions [A1] to [A3] become:
$[A1']$ For $i = 1, 2$, A_i, C_i are as A_0, C_0 in (2), $\varphi_i(x, u) =$

$$(\varphi_{i1}(x_1[i], u), \;.. \quad \varphi_{in_i-1}(x_1[i], ..x_{n_i-1}[i], u), \; \varphi_{in_i}(x[i], x[j], u))^T, j \neq i,$$

and $\psi_i(x, u)$ is as in (7).
$[A2']$ The same as [A2];
$[A3']$ Given some K_i making $A_i - K_iC_i$ to be a stable matrix for $i = 1, 2$, inputs u are such that the state vector satisfies the following property:
for any $x(0) \in \chi_0$, and any $\Gamma_i(0) \in I\!R^{n_i \times q}$, the solutions $\Gamma_i(t)$ of:

$$\dot{\Gamma}_i = \lambda_i(A_i - K_iC_i)\Gamma_i + \lambda_i\psi_i(x, u) \qquad (22)$$

is such that for some $t_0 \geq 0$:

$$\begin{array}{l} \exists \alpha_i, T_i \text{ independent of } \lambda_i \;:\; \forall t \geq t_0, \\ \text{and for } \lambda_i \text{ large enough,} \\ \displaystyle\int_t^{t+T_i} \Gamma_i(\tau)^T C_i^T C_i \Gamma_i(\tau) d\tau \geq \alpha_i I. \end{array} \qquad (23)$$

Then by similar arguments as in previous subsection, one can prove the following:

Theorem *3.2.* Given a system (21) satisfying assumptions $[A1'], [A2'], [A3']$, for λ large enough, the system below is an asymptotic observer for (21), in the sense that for any initial condition $x(0) \in \chi_0$ and any $\hat{\theta}(0)$, $\hat{x}(0)$ respectively bounded by Θ and X, $\|\hat{x}(t) - x(t)\|$ and $\|\hat{\theta}(t) - \theta\|$ exponentially go to zero:

$$\begin{array}{l} \dot{\hat{\Gamma}}(t) = \lambda(A - KC)\hat{\Gamma}(t) + \lambda\psi(\tilde{x}(t), u(t)) \\ \dot{\hat{x}}(t) = A\hat{x}(t) + \varphi(\tilde{x}(t), u(t)) + \psi(\tilde{x}(t), u(t))\tilde{\theta}(t) \\ \qquad + \Lambda(\lambda)^{-1}[\lambda K + \hat{\Gamma}(t)\hat{\Gamma}^T(t)C^T][y(t) - C\hat{x}(t)] \\ \dot{\hat{\theta}}(t) = I_\lambda \hat{\Gamma}(t)^T C^T [y(t) - C\hat{x}(t)] \\ \quad \tilde{x} = \hat{x} \; if \; \|\hat{x}\| \leq X, \; \dfrac{\hat{x}}{\|\hat{x}\|} X \; otherwise, \\ \quad \tilde{\theta} = \hat{\theta} \; if \; \|\hat{\theta}\| \leq \Theta, \; \dfrac{\hat{\theta}}{\|\hat{\theta}\|} \Theta \; otherwise. \end{array} \qquad (24)$$

with $K = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$, $\Lambda = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}$, Λ_i as in (4), K_i making $A_i - K_iC_i$ to be stable, and $I_\lambda = \begin{pmatrix} \lambda^{n_1} I & 0 \\ 0 & \lambda^{n_2} I \end{pmatrix}$.

Lemma 3.1 indeed clearly applies to each subvector $x[i]$ of x.
Then notice that by choosing $\lambda = max(\lambda_1, \lambda_2)$, $\Gamma = \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix}$ with Γ_i as in (22), satisfies a property (23), and with similar arguments as in the proof of lemma 3.2, so can do $\hat{\Gamma}$ of (24).
Finally, by a similar transformation as in the proof of lemma 3.3 (with here $\varepsilon_\theta = I_\lambda^{-1}(\hat{\theta} - \theta)$), one still obtains error equations of the form (20), and by combining Lyapunov functions respectively associated to ε and ε_θ the conclusion follows in the same way.

4. ILLUSTRATIVE EXAMPLES

As a first example, let us consider the single output system described by:

$$\begin{array}{rl} \dot{x}_1 &= x_2 + u \\ \dot{x}_2 &= -x_1 - 2sin(x_2) + arctan(x_2)\theta_1 \\ & \quad +cos(x_1x_2)\theta_2 \\ y &= x_1 \end{array}$$

where $u(t) = sin(2t) + \frac{1}{10}cos(10t)$, $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
Here the system is clearly under the form (9) and thus one can design an observer (12). Results obtained with $\lambda = 10$ are shown by figure 1 for state estimation, and figure 2 for parameter estimation. From those figures, it can be seen that the expected estimation is achieved.

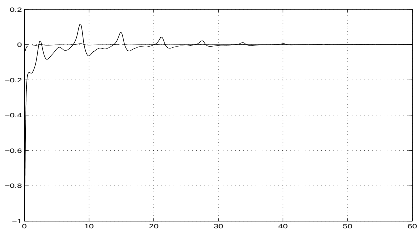

Fig. 1. State estimation errors ($\hat{x}_1 - x_1$, $\hat{x}_2 - x_2$).

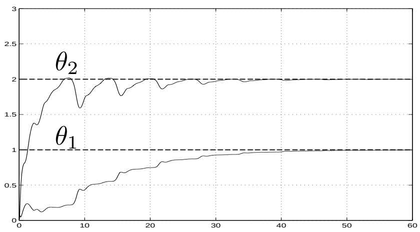

Fig. 2. Parameter estimation($\hat{\theta}_1$ vs θ_1, $\hat{\theta}_2$ vs θ_2).

As a second example, let us consider the problem of identifying parameters in a system represented by the following input-output relationship:

$$y^{(2)} = sin(y\dot{y}) + cos(y\dot{u})\theta_1 + \dot{u}u^2\theta_2.$$

Here by considering for instance some piecewise linear approximation of the previous input function $u(t) = sin(2t) + \frac{1}{10}cos(10t)$, the system can be written as (6) (with $x[1] = \begin{pmatrix} y \\ \dot{y} \end{pmatrix}$ and $x[2] = \begin{pmatrix} u \\ \dot{u} \end{pmatrix}$).

An observer (24) can then be designed so as to get estimations of θ_1, θ_2 (and $\dot{y}, \dot{u}$) from the only measurement of y, u.
Simulation results obtained with $\lambda = 40$ are given in figure 3 (where $\theta_1 = 10, \theta_2 = -5$).

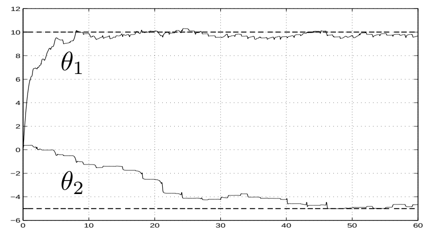

Fig. 3. Parameter identification ($\hat{\theta}_1$ vs θ_1, $\hat{\theta}_2$ vs θ_2).

5. CONCLUSION

In this paper, an adaptive version of the well-known high gain observer for nonlinear systems has been proposed, as an extension of previous results of (Zhang, 2002; Zhang *et al.*, 2003) in two directions: the unknown parameters enter the system through state-dependent functions on the one hand, and a particular multi-output case has been considered, motivated by a possible state-space representation of nonlinear input-output models.

REFERENCES

Bastin, G. and M.R. Gevers (1988). Stable adaptive observers for nonlinear time-varying systems. *IEEE Trans. on Automatic Control* **33**(7), 650–658.

Besançon, G. (2000). Remarks on nonlinear adaptive observer design. *Systems & Control Letters* **41**, 271–280.

Besançon, G. (2003). High gain observation with disturbance attenuation and application to robust fault detection. *Automatica* **39**, 1095–1102.

Besançon, G. and H. Hammouri (1998). On observer design for interconnected systems. *Journal of Mathematical Systems, Estimation, & Control*.

Besançon, G. and Q. Zhang (2002). Further developments on nonlinear adaptive observers with application to fault detection. In: *IFAC World Congress, Barcelona, Spain*.

Bornard, G. and H. Hammouri (1991). A high gain observer for a class of uniformly observable systems. In: *Proc. 30th IEEE Conf. on Decision and Control, Brighton, England, USA*. pp. 1494–1496.

Bornard, G. and H. Hammouri (2002). A graph approach to uniform observability of nonlinear multi-output systems. In: *41st IEEE Conf. on Decision and Control, Las Vegas, USA*. pp. 701–706.

Carroll, R.L. and D.P. Lindorff (1973). An adaptive observer for single input single output linear systems. *IEEE Trans. on Automatic Control* **18**(5), 428–435.

Cho, Y.M. and R. Rajamani (1997). A systematic approach to adaptive observer synthesis for nonlinear systems. *IEEE Trans. on Automatic Control* **42**(4), 534–537.

Ciccarella, G., M. Dalla Mora and A. Germani (1993). A luenberger-like observer for nonlinear systems. *Int. Journal of Control* **57**, 537–556.

Gauthier, J.P. and G. Bornard (1981). Observability for any $u(t)$ of a class of nonlinear systems. *IEEE Trans. on Automatic Control* **26**(4), 922–926.

Gauthier, J.P., H. Hammouri and S. Othman (1992). A simple observer for nonlinear systems - applications to bioreactors. *IEEE Trans. on Automatic Control* **37**(6), 875–880.

Kreisselmeier, G. (1977). Adaptive observers with exponential rate of convergence. *IEEE Trans. on Automatic Control* **22**, 2–8.

Ljung, L. and T. Glad (1994). On global identifiability for arbitrary model parametrizations. **30**, 265–276.

Lüders, G. and K.S. Narendra (1973). An adaptive observer and identifier for a linear system. *IEEE Trans. on Automatic Control* **18**(5), 496–499.

Marino, R. (1990). Adaptive observers for single output nonlinear systems. *IEEE Trans. on Automatic Control* **35**(9), 1054–1058.

Martinez, J. Correa and A. Poznyak (2001). Switching structure robust state and parameter estimator for MIMO nonlinear systems. *Int. J. Control* **74**(2), 175–189.

Narendra, K.S. and A. Annaswamy (1989). *Stable Adaptive Systems*. Prentice Hall Int., NJ.

Sastry, S. and M. Bodson (1989). *Adaptive control - stability, convergence and robsutness*. Prentice Hall Int., NJ.

Xu, A. (2002). Observateurs adaptatifs non linéaires et diagnostic de pannes. PhD thesis. Université de Rennes I.

Zhang, Q. (2002). Adaptive observer for MIMO linear time-varying systems. *IEEE Trans. Auto. Control* **47**(3), 525–529.

Zhang, Q., A. Xu and G. Besançon (2003). An efficient nonlinear adaptive observer with global convergence. In: *13th IFAC Symp. on System Identification, Rotterdam, The Netherlands*.